The Emergence of Classicality via Decoherence Described by Lindblad Operators

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Abstract

Zurek, Habib and Paz [W. H. Zurek, S. Habib and J. P. Paz, Phys. Rev. Lett. **70** (1993) 1187] have characterized the set of states of maximal stability defined as the set of states having minimum entropy increase due to interaction with an environment, and shown that coherent states are maximal for the particular environment model examined. To generalize these results, I consider entropy production within the Lindblad theory of open systems, treating environment effects perturbatively. I characterize the maximally predicitive states which emerge from several forms of effective dynamics, including decoherence from spatially correlated noise. Under a variety of conditions, coherent states emerge as the maximal states.

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I. INTRODUCTION

Decoherence which results from a quantum system's interaction with an environment can provide a mechanism for characterizing the transition from quantum to classical behavior for a quantum open system [1,2], and has been an integral part of several programs addressing the emergence of classicality [3,4]. Zurek, Habib and Paz (ZHP) have characterized the effectiveness of decoherence in terms of a predictability sieve, and identified the maximally predictive states (defined as those with minimal entropy production) as the most classical [5]. ZHP considered an environment model consisting of an independent oscillator bath linearly coupled to the system of interest, which has been studied in the context of quantum brownian motion [6]. ZHP demonstrated for the high temperature limit of the environment that the coherent states of a harmonic oscillator are maximally predictive, and that zero squeezing (corresponding to coherent states) is maximal for squeezed states considered at arbitrary environment temperatures.

The purpose of this paper is to extend the results of ZHP to additional environment models. The effects of the environment are considered in the general framework of the Lindblad form for nonunitary evolution of a harmonic oscillator [7], correponding to the Markov limit. In Sec. II I establish a systematic framework for evaluating the predictability of states using first order perturbation theory. In Sec. III I apply this framework to a family of Lindblad generators which are at most quadratic in position and momentum whose general properties have been studied in the literature [8]. This family of Lindblad generators includes as special cases the Quantum Optical Master Equation [9] and Dekker's model for Quantum dissipation [10]. I consider models with environment spatial correlation effects [11,12] in Sec. IV. I comment on these results in Sec. V.

II. PERTURBATIVE APPROACH TO EVALUATING PREDICTABILITY

The predictability sieve was introduced by Zurek [3] as a means of characterizing those states which are most stable when considering not only the dynamics of the system, but also including the effects (such as decoherence) of interaction with the systems environment. The set of states having minimum (linear) entropy production are the best candidates for corresponding to points of classical phase space. The linear entropy ς for a particular density operator ρ is given by:

$$\varsigma(t) = \text{Tr}[\rho(t) - \rho(t)^2] = 1 - \text{Tr}[\rho(t)^2]. \tag{2.1}$$

The effect of the environment on evolution is taken to be in the Markov regime, and I assume that there is no further explicit time dependence on the Liouville generator for the isolated system. The form of the evolution is then stationary, and we can write

$$\rho(t) = e^{Lt}[\rho] = J(t)[\rho]. \tag{2.2}$$

The generator L is assumed to be comprised of two parts, the evolution of the isolated system corresponding to L_o with $J_o(t) = e^{L_o t}$, and the effects of the environment ΔL. I consider the effects of the environment perturbatively in part because a wide variety of forms of evolution become tractable, and also because many of the effective evolution equations are derived using lowest order approximations such as the weak coupling limit [13]. From formal perturbation theory, the evolution of Liouville generator can be written

$$\begin{aligned} J(t) &= J_o(t) + \int_0^t J_o(t-\tau)\Delta L J(\tau)d\tau \\ &\cong J_o(t) + \int_0^t J_o(t-\tau)\Delta L J_o(\tau)d\tau + O(\Delta L^2) \\ &= J_o(t) + J_1(t) + O(\Delta L^2), \end{aligned} \tag{2.3}$$

defining $J_1(t)$ as the first order perturbation. For an initial state ρ_o, the entropy at time t becomes

$$\varsigma(t) = 1 - \text{Tr}[J(t)[\rho_o]J(t)[\rho_o]]$$

$$\cong 1 - \text{Tr}[J_o(t)[\rho_o]J_o(t)[\rho_o]]$$

$$-2\text{Tr}[J_o(t)[\rho_o]J_1(t)[\rho_o]]. \tag{2.4}$$

If the isolated system has unitary evolution generated by a hamiltonian H, then

$$L_o[\rho] = \frac{1}{i\hbar}[H, \rho],$$

$$J_o[\cdot] = U(t) \cdot U^\dagger(t),$$

$$U(t) = e^{\frac{1}{i\hbar}Ht}. \tag{2.5}$$

In this case

$$1 - \text{Tr}[J_o(t)[\rho_o]J_o(t)[\rho_o]] = 1 - \text{Tr}[\rho_o\rho_o] = \varsigma(0). \tag{2.6}$$

The maximal states are those with minimal entropy production, that is, with minimized

$$\Delta\varsigma(t) \equiv -2\text{Tr}[J_o(t)[\rho_o]J_1(t)[\rho_o]]$$

$$= -2\int_0^t \{\text{Tr}[U(t)\rho_o U^\dagger(t)U(t-\tau)\Delta L[U(s)\rho_o U^\dagger(s)]U^\dagger(t-\tau)]\}d\tau$$

$$= -2\int_0^t \text{Tr}[\rho_o U^\dagger(\tau)\Delta L[U(\tau)\rho_o U^\dagger(\tau)]U(\tau)]d\tau, \tag{2.7}$$

where the cyclic property of the trace and the unitarity of $U(t)$ have been used for the final simplification.

The general form for a Lindblad generator is

$$L[\rho] = \frac{1}{i\hbar}[H, \rho] + \frac{1}{2\hbar}\sum_j [V_j\rho, V_j^\dagger] + [V_j, \rho V_j^\dagger], \tag{2.8}$$

so that the contribution to environment interaction can be written as the perturbation

$$\Delta L[\rho] = \frac{1}{i\hbar}[\Delta H, \rho] + \frac{1}{2\hbar}\sum_j [V_j\rho, V_j^\dagger] + [V_j, \rho V_j^\dagger]. \tag{2.9}$$

The entropy production can now be written

$$\Delta\varsigma(t) = -2\int_0^t \text{Tr}[\rho_o U^\dagger(\tau)\frac{1}{i\hbar}[\Delta H, U(\tau)\rho_o U^\dagger(\tau)]U(\tau)d\tau$$

$$-\frac{1}{\hbar}\int_0^t \text{Tr}[\rho_o U^\dagger(\tau)\sum_j ([V_j U(\tau)\rho_o U^\dagger(\tau), V_j^\dagger]$$

$$+[V_j, U(\tau)\rho_o U^\dagger(\tau)V_j^\dagger])U(\tau)]d\tau. \tag{2.10}$$

The first term on the RHS of this equation is identically zero, from the cyclic property of the trace. The remainder can written in a simpler form using the cyclic property of the trace, the unitarity of $U(t)$ and the identification:

$$V_j(\tau) \equiv U^\dagger(\tau)V_j U(\tau) \tag{2.11}$$

to yield

$$\Delta \varsigma(t) = \frac{1}{\hbar} \int_0^t \sum_j \mathrm{Tr}[\{V_j^\dagger(\tau)V_j(\tau), \rho_o\}$$
$$-2V_j(\tau)\rho_o V_j^\dagger(\tau)]d\tau. \tag{2.12}$$

If ρ_o is a pure state, then it is also a projection, with

$$\rho_o = P = |\psi\rangle\langle\psi| \tag{2.13}$$

with

$$P^2 = P \tag{2.14}$$

and

$$POP = P\langle\psi|O|\psi\rangle = P\langle O\rangle \tag{2.15}$$

for an arbitrary operator O. Thus, for pure states, the entropy production is given by

$$\Delta \varsigma(t) = \frac{2}{\hbar} \int_0^t \sum_j (\langle\{V_j^\dagger(\tau)V_j(\tau)\rangle - \langle V_j(\tau)\rangle\langle V_j^\dagger(\tau)\rangle)d\tau. \tag{2.16}$$

Minimization of this final quantity can then be used to determine the maximal states.

III. LINDBLAD OPERATORS QUADRATIC IN POSITION AND MOMENTUM

I will now apply the results of Sec. II to Lindblad operators which have $\{V_j\}$ linear in position and momentum. This family of generators have been studied extensively in the literature [8], and include as special cases the quantum optical master equation [9] and

Dekker's phenomenological master equation [10]. In terms of Eq. (2.9), the operators are given by

$$\Delta H = \frac{\mu}{2}\{x,p\}$$

$$V_j = a_j p + b_j x. \tag{3.1}$$

With the identifications

$$D_{qq} = \frac{\hbar}{2}\sum_j |a_j|^2,$$

$$D_{pp} = \frac{\hbar}{2}\sum_j |b_j|^2,$$

$$D_{pq} = \frac{\hbar}{2}\sum_j -\mathrm{Re}[a_j b_j^*],$$

$$\lambda = \sum_j \mathrm{Im}[a_j b_j^*], \tag{3.2}$$

the perturbation on the system evolution becomes

$$\Delta L[\rho] = \frac{1}{i\hbar}[\frac{\mu}{2}\{x,p\},\rho] - \frac{D_{qq}}{\hbar^2}[p,[p,\rho]] - \frac{D_{pp}}{\hbar^2}[x,[x,\rho]]$$
$$+ \frac{D_{pq}}{\hbar^2}([x,[p,\rho]] + [p,[x,\rho]])$$
$$+ \frac{i\lambda}{2\hbar}([x,\{p,\rho\}] - [p,\{x,\rho\}]). \tag{3.3}$$

The particular choice of parameters $D_{qq}, D_{pp}, D_{pq}, \lambda$ and μ Eq. (3.3) determines the details of the evolution (i.e. evolution corresponding to Quantum Optical Master Equation, Dekker's master equation, etc.). It has been determined that only when

$$D_{pq} = \frac{\hbar}{2}\sum_j -\mathrm{Re}[a_j b_j^*] = 0 \tag{3.4}$$

will the system relax into a thermal equilibrium state [8], so I adopt this condition for the remainder of the paper.

For the simple harmonic oscillator, the operator equations of motion are easily solved by

$$x(\tau) = U^\dagger(\tau) x U(\tau)$$

$$= x\cos(\omega\tau) + \frac{p}{m\omega}\sin(\omega\tau),$$

$$p(\tau) = U^\dagger(\tau) p U(\tau)$$

$$= p\cos(\omega\tau) - m\omega x \sin(\omega\tau), \tag{3.5}$$

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to yield

$$V_j(\tau) = a_j(p\cos(\omega\tau) - m\omega x \sin(\omega\tau))$$

$$+ b_j(x\cos(\omega\tau) + \frac{p}{m\omega}\sin(\omega\tau)). \tag{3.6}$$

Substituting Eq. (3.6) into Eq. (2.16) and evaluating the elementary trigonometric integrals over τ:

$$\Delta_\varsigma(t) = f_1(t)(\frac{1}{2m}\langle p^2 - \langle p\rangle^2\rangle + \frac{m\omega^2}{2}\langle x^2 - \langle x\rangle^2\rangle) - 2\lambda t$$

$$+ f_2(t)(\frac{1}{2m}\langle p^2 - \langle p\rangle^2\rangle - \frac{m\omega^2}{2}\langle x^2 - \langle x\rangle^2\rangle)$$

$$+ f_3(t)(\frac{\omega}{2}\langle\{x,p\}\rangle - \omega\langle x\rangle\langle p\rangle), \tag{3.7}$$

where

$$f_1(t) = t2m(\frac{2D_{qq}}{\hbar^2} + \frac{2D_{pp}}{(m\omega\hbar)^2}),$$

$$f_2(t) = 2m\frac{\sin(2\omega t)}{2\omega}(\frac{2D_{qq}}{\hbar^2} - \frac{2D_{pp}}{(m\omega\hbar)^2}),$$

$$f_3(t) = -2\frac{\sin^2(\omega t)}{\omega^2}(\frac{2D_{qq}}{\hbar^2} - \frac{2D_{pp}}{(m\omega\hbar)^2}). \tag{3.8}$$

Eq. (3.7) is the expectation of a c-number times the harmonic oscillator hamiltonian plus a second c-number constant after sqeezeing and translation by $\langle x\rangle$ in position and $\langle p\rangle$ in momentum. The state which minimizes Eq. (3.7) will be the corresponding squeezed and translated ground state, which is simply a coherent squeezed state [14]. In terms of the harmonic oscillator creation and annihilation operators $a^\dagger$ and a, the squeeze operator is given by

$$S(\zeta) = e^{\frac{1}{2}(\zeta^* a^2 - \zeta a^{\dagger 2})}, \tag{3.9}$$

and Glauber's displacement operator is given by

$$D(\alpha) = e^{(\alpha a^\dagger - \alpha a)}. \tag{3.10}$$

The appropriate selection of the paramter α for the displacement operator

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$$\mathrm{Im}(\alpha) = (\frac{\hbar}{2m\omega})^{\frac{1}{2}}\langle p \rangle,$$

$$\mathrm{Re}(\alpha) = (\frac{\hbar m\omega}{2})^{\frac{1}{2}}\langle x \rangle, \tag{3.11}$$

provides the necessary translation (in phase space). For simplicity, I take $\langle x \rangle = 0$ and $\langle p \rangle = 0$ for the rest of this section. The effect of squeezing on the anihillation operator can be written

$$S^\dagger(\zeta)aS(\zeta) = \mu a + \nu a^\dagger, \tag{3.12}$$

where

$$\zeta = se^{i\theta},$$

$$\mu = \cosh(s),$$

$$\nu = \sinh(s)e^{i\theta}. \tag{3.13}$$

The parameter s determines the amount of squeezing ($s = 0$ corresponding to no squeezing), and the parameter θ determines the orientation of the sqeeze axis. The squeezed harmonic oscillator hamiltonian is given by

$$\begin{aligned}
S^\dagger(\zeta)(\frac{p^2}{2m} &+ \frac{m\omega^2}{2}x^2)S(\zeta) \\
&= \cosh(2s)(\frac{m\omega^2}{2}x^2 + \frac{p^2}{2m}) \\
&+ \sinh(2s)\sin(\theta)(\frac{m\omega^2}{2}x^2 - \frac{p^2}{2m}) \\
&+ \cosh(2s)\cos(\theta)\frac{\omega}{2}\{x, p\},
\end{aligned} \tag{3.14}$$

which can be used to identify the amount and direction of squeezing required to minimize Eq. (3.7). The easiest condition to extract is the direction of squeezing:

$$\cos(\theta) = \frac{f_3(t)}{f_1(t)}. \tag{3.15}$$

Since $f_1(t)$ is linear in t while $f_3(t)$ oscillates, the long time behavior is $\theta = 0$. The amount of squeezing can be determined by examining

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$$(1 + \cosh^2(2s)) \sin(\theta) = \frac{f_2(t)}{f_1(t)} \cosh(2s). \tag{3.16}$$

To analyze the long time behavior, it is useful to note that $\sin(\theta)$ approaches 1 and $f_2(t)/f_1(t)$ approaches 0 (and therefore so does the RHS of Eq. (3.16)). Thus, the long time behavior of the amount of squeezing required for minimum entropy production is

$$(1 + \cosh^2(2s)) = 0, \tag{3.17}$$

which implies $s = 0$, no squeezing. Thus, for times on the order of the dynamical time scale of the system (more than a few cycles), coherent states are the maximal states, just as ZHP found for their environment model.

IV. CORRELATION EFFECTS IN ENVIRONMENT NOISE

In this section I wish to consider environment models which include the effects of finite correlation lengths in the environment [11,12]. From a strictly phenomenological point of view, Quantum Mechanics with Spontaneous Localization can be included by virtue of the effective form of the dynamics, although this is actually a fundamental modification of Quantum Mechanics [15]. Many of these models can be written in the form

$$\frac{\partial \rho(x, x'; t)}{\partial t} = \text{Hamiltonian} + \text{Dissipation}$$
$$+ \cdots - g(x, x')\rho(x, x'; t), \tag{4.1}$$

where the decoherence term which we shall focus upon can be expressed in terms of the correlations of a classical fluctuating potential $V(x, t)$ with

$$g(x; y) = \frac{1}{\hbar^2}(c(x; x) + c(y; y) - 2c(x; y)), \tag{4.2}$$

where

$$\langle V(x, t)V(y, \tau) \rangle_{\text{av}} = c(x; y)\delta(t - \tau). \tag{4.3}$$

For simplicity, I consider only a homogeneous and isotropic environment for which $c(x; y) = c(x - y)$, and $g(x; y) = g(x - y)$. I will also restrict my attention to weak dissipation

, and consider only the evolution due to the unperturbed hamiltonian and the (spatially correlated) noise term.

The Lindblad form can be used to represent the noise term with

$$\{V_j\} = a(k)e^{ikx} \tag{4.4}$$

and replacing the discrete sum over j in Eq. (2.9) with an integral over k. The evolution in this case can be written

$$\frac{\partial \rho(x, x'; t)}{\partial t} = \text{Hamiltonian} - \frac{1}{\hbar} \int dk |a(k)|^2 (1 - e^{ik(x-x')}) \rho(x, x'; t), \tag{4.5}$$

so that $|a(k)|^2$ and $c(r)$ are Fourier transform pairs:

$$c(r) = \frac{\hbar}{2} \int |a(k)|^2 e^{ik(r)} dk. \tag{4.6}$$

Thus a noise term with short correlation length scales will have a narrow $c(r)$ and a broad $|a(k)|^2$ while a long correlation length scale implies a narrow $|a(k)|^2$. Inserting Eq. (4.4) into Eq. (2.16) yields

$$\Delta_\varsigma(t) = \frac{1}{\hbar} \int_0^t \int |a(k)|^2 (1 - |\langle e^{ikx(\tau)} \rangle|^2) dk d\tau, \tag{4.7}$$

where $x(\tau)$ is given by Eq. (3.5), and $e^{ikx(s)}$ can immediately be recognized as Glauber's displacement operator, with a translation in momentum of $\hbar k \cos(\omega\tau)$ and a translation in position of $-(\hbar k)/(m\omega) \sin(\omega\tau)$. In terms of the translated state

$$|\psi; k, \tau\rangle \equiv e^{ikx(\tau)} |\psi\rangle, \tag{4.8}$$

the entropy production is given by

$$\Delta_\varsigma(t) = \frac{1}{\hbar} \int_0^t \int |a(k)|^2 (1 - |\langle \psi | \psi; k, \tau \rangle|^2) dk d\tau. \tag{4.9}$$

While it is not possible to find general solutions for the minimization of Eq. (4.9) for arbitrary environment correlations, it is possible to extract important limiting cases. Restricting attention to times of several oscillator periods or more, entropy minimization requires the

maximization of $|\langle\psi|\psi;k,\tau\rangle|^2$ for typical values of k, on the order of δk (the spread of $|a(k)|^2$), resulting in typical translations of $\hbar\delta k$ in momentum and $(\hbar\delta k)/(m\omega)$ in position. The maximization of the square of the inner product of any two normalized vectors occurs when the vectors are identical (up to a phase). Thus the maximal states and the translated maximal states will be approximately equal for typical translations, requiring that the width of $|a(k)|^2$ be much less than the width of the maximal states:

$$\delta k \ll \frac{m\omega}{\hbar}\Delta x$$
$$\delta k \ll \frac{1}{\hbar}\Delta p. \tag{4.10}$$

Since $|a(k)|^2$ and $c(r)$ are Fourier transform pairs, this last condition also implies that the noise spatial correlation function $c(r)$ is wide (compared to the maximal states). In this long correlation length scale limit, $g(x,x')$ in Eq. (4.1) is quadratic [11], corresponding to the low dissipation limit studied by ZHP and to the results of Sec. III with $b_j = 0$ for all j. The coherent states are then the maximal states if they are consistent with the condition expressed in Eq. (4.10) using coherent state values for Δx and Δp:

$$\delta k \ll \sqrt{\frac{m\omega}{2\hbar}}, \tag{4.11}$$

which corresponds to a environment corellatation length much larger than the width of a coherent state $\Delta x = \sqrt{\hbar/2m\omega}$.

If the environment correlation length is shorter than the width of the coherent state, then the approximation described above is not valid. It is useful to examine the entropy production in the position representation and absorbing the time dependance into the (Schrödinger picture) state vectors, where

$$\langle\psi|e^{ikx(\tau)}|\psi\rangle = \langle\psi|U^\dagger(\tau)e^{ikx}U(\tau)|\psi\rangle$$
$$= \langle\psi(\tau)|e^{ikx}|\psi(\tau)\rangle$$
$$= \int dx\, e^{ikx}|\psi(x,\tau)|^2$$
$$= \int dx\, e^{ikx}P(x,\tau). \tag{4.12}$$

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Entropy production then becomes

$$\Delta_\varsigma(t) = \frac{2}{\hbar^2}[c(0)t - \int d\tau dx dx' c(x - x')P(x,\tau)P(x',\tau)], \qquad (4.13)$$

using the Fourier transform relation expressed in Eq. (4.6). For narrow $c(r)$ this expression becomes independent of the function P. To see this, consider an example where the spatial correlation of the environment is given by

$$c(r) = \lambda e^{(\frac{r}{\sigma})^2}. \qquad (4.14)$$

In the limit σ approaches zero

$$\int dx dx' \lambda e^{(\frac{x-x'}{\sigma})^2} P(x,\tau)P(x',\tau) \cong$$
$$\lambda \sigma \sqrt{\pi} \int P^2(x,\tau)dx \to 0, \qquad (4.15)$$

so that in the short correlation length limit

$$\Delta_\varsigma(t) = \frac{2}{\hbar^2}c(0)t \qquad (4.16)$$

so that in this regime, all states produce the same entropy, there are no maximal states. However, the decay rate of the off diagonal terms given by the decoherence term in Eq. (4.1) in the low short correlation length regime is generally at a maximum,

$$\frac{\partial \rho(x,x';t)}{\partial t} = \cdots - \frac{2c(0)}{\hbar^2}\rho(x,x';t), \qquad (4.17)$$

and if decoherence is to be effective, the decay time must be comparable to dynamical timescales. There will necessarily be a significant increase in the entropy for all pure states, so that all states will be rapidly ` mixed´ by the noise.

V. COMMENTS AND CONCLUSIONS

I have established an approximation scheme for determining maximal states (as defined by ZHP), and applied it to two families of Lindblad operators. For Lindblad operators

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which are at most quadratic in position and momentum, squeezed states emerge as the maximal states for intermediate times compared to the dynamical time scales. The amount of squeezing decreases with time, so that coherent states are maximal for large timescales. Large timescales are the most relevant, since an object's classicality should be an enduring property, not a transient one. For an environment with finite spatial correlation, coherent states emerge as maximal when the environment has long correlation length, but all states rapidly become mixed states when the environment length scale is long. Thus environment correlation effects will not be important in establishing the nature of maximal states and the character of quasiclassical states. However, correlation effects can still be important when considering quantum interference between two such states. One important result which emerges from these calculations is that coherent states are a robust choice for the maximal states.

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